I B. Tech II Semester Regular/Supplementary Examinations, August- 2022

MATHEMATICS-III

(Only EEE)

Time: 3 hours Max. Marks: 70

Answer any five Questions one Question from Each Unit
All Questions Carry Equal Marks

Unit - I

1. a) Prove that $\nabla\left(\frac{1}{r}\right)=\frac{-\bar{r}}{r^3}$ (7M)

b) Using Stoke's theorem to evaluate $\oint_c \bar{A}.d\bar{r}$ where $\bar{A}=2y^2\,\bar{i}+3x^2\bar{j}-(2x+z)\bar{k}$ and c is the boundary of the triangle whose vertices are (0, 0, 0), (2, 0, 0), (2, 2, 0). (7M)

Or

2. a) Prove that $div\,\left(\bar{A}\times\bar{B}\right)=\bar{B}.curl\,\bar{A}-\bar{A}.curl\,\bar{B}$ (7M)

b) **If** $\bar{F}=(x-y)\,\bar{i}+(x+y)\,\bar{j}$ Evaluate $\oint_c \bar{F}.d\bar{r}$ around the curve $y=x^2$ and $y^2=x$ (7M)

Unit - II

3. a) Use theorem on Laplace transform of derivatives to evaluate $L\left\{\frac{cos\sqrt{t}}{\sqrt{t}}\right\}$ (7M)

b) Find inverse Laplace transform of $\left\{\frac{s}{(s^2+a^2)^2}\right\}$ using convolution theorem. (7M)

Or

4. a) Find$L\{f(t)\}$where $f(t)$ is a periodic function of period 2π and is given by (7M)

$$f(t)=\begin{cases} sint, & 0<t<\pi \\ 0, & \pi\le t<2\pi \end{cases}$$

b) Find $L^{-1}\left\{\frac{s^2+2}{s\,(s^2+4)}\right\}$ (7M)

Unit - III

5. a) Find the Fourier series of $f(x)=\begin{cases} 1+\frac{2x}{\pi} & if\ -\pi\le x<0 \\ 1-\frac{2x}{\pi} & if\ 0\le x<\pi \end{cases}$ (7M)

Hence deduce that $1+\frac{1}{3^2}+\frac{1}{5^2}+......=\frac{\pi^2}{8}$

b) Find the Fourier transform of $f(x)$ defend by $f(x)=\begin{cases} x & if\ |x|<a \\ 0 & if\ |x|>a \end{cases}$ (7M)

Or

6. a) Find the Half range cosine series of $f(x)=e^x$ in [0,1] (7M)

b) Find inverse Fourier sine transform of $\frac{1}{p}e^{-ap}$ (7M)

Unit - IV

7. a) Solve the PDE $p^{1/3} - q^{1/3} = 3x - 2y$ (7M)

b) Find the partial differential equation by eliminating arbitrary function from $ax + by + cz = \phi(x^2 + y^2 + z^2)$ (7M)

Or

8. a) From the partial differential equation of all planes which are at a constant distance 'a' from the origin. (7M)

b) Solve the PDE $(mz - ny)p + (nx - lz)q = ly - mx$ (7M)

Unit - V

9. a) Solve the partial differential equation $\left(D^2 - 2DD^1 + D^{1^2}\right)z = \sin(2x + 3y)$ (7M)

b) Solve the PDE $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$ and $u(x, y) = 4e^{-x}$ by the method of variation of parameters. (7M)

Or

10. a) Solve the partial differential equation $\left(D^2 - D^{1^2}\right)z = x^2 + y^2$ (7M)

b) A rectangular plate with insulated surface is 8 cm. wide. If the temperature along one short edge y = 8 cm. is given by $100\sin\frac{\pi x}{8}$, 0 < x < 8 while the two long edges x = 0 and x = 8 and other edge are kept 0^0c .Find the steady state temperature at any point on the plane . (7M)

Code No: R201206 **R20** SET - 2

I B. Tech II Semester Regular/Supplementary Examinations, August- 2022

MATHEMATICS-III

(Only EEE)

Time: 3 hours Max. Marks: 70

Answer any five Questions one Question from Each Unit
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Unit - I

1 a) Show that $\bar{F} = (e^x z - 2xy)\,\bar{i} - (x^2 - 1)\bar{j} + (e^x + z)\,\bar{k}$ is conservative field, Hence Evaluate $\oint_c \bar{F}.d\bar{r}$ where c is the end point of the curve (0, 1,–1) and (2, 3, 0). (7M)

b) Prove that $\nabla^2\left(\frac{1}{r}\right) = 0$ (7M)

Or

2 a) Prove that $\nabla\left(e^{r^2}\right) = 2e^{r^2}\,\bar{r}$ (7M)

b) Verify Green's theorem for $\int_c (3x^2 - 8y^2)dx + (4y - 6xy)\,dy$ where c is the boundary of the region enclosed by the lines. $x = 0$ $y = 0$ $x + y = 1$. (7M)

Unit - II

3 a) Find $L\left\{\frac{sin^2 t}{t^2}\right\}$ (7M)

b) Using Laplace transform method, solve the following differential equation $(D^2 + D)x = 2$ if $x(0) = 3, x'(0) = 1, x''(0) = -2$ (7M)

Or

4 a) Find Laplace transform of unit impulse function or Dirac delta function (7M)

b) Find inverse Laplace transform of $\left\{\frac{1}{(s-2)(s+2)^2}\right\}$ using convolution theorem. (7M)

Unit - III

5 a) Find the Fourier series of $f(x) = x + x^2 \quad in \quad (-\pi, \pi)$ (7M)

b) If F(p) is the complex Fourier transform of f(x), then prove that the complex Fourier transform of f(x) cos ax is $\frac{1}{2}\left[F(p+a) + F(p-a)\right]$ (7M)

Or

6 a) Find the Half range sine series of $f(x) = \begin{cases} \frac{1}{4} - x & 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \frac{1}{2} < x < 1 \end{cases}$ (7M)

b) Find the Fourier sine transform of the function f(x) = x (7M)

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Unit - IV

7 a) Find the partial differential equation by eliminating arbitrary constants a & b from $z = a\log\left[\frac{b(y-1)}{1-x}\right]$ (7M)

b) Solve the PDE $q = px + p^2$ (7M)

Or

8 a) Find the partial differential equation by eliminating arbitrary functions from $z = f(2x+3y) + yg(2x+3y)$ (7M)

b) Solve the PDE $(x + 2z)p + (4z - y)q = 2x + y$ (7M)

Unit - V

9 a) Solve the partial differential equations $\left(D^3 - D^{1^3}\right)z = x^3y^3$ (7M)

b) Find the temperature u(x, t) in a homogenous bar of heat conducting method of length 'l' whose ends are kept at 0^0c and whose initial temperature is $\frac{ax}{l^2}(l-x)$ (7M)

Or

10 a) Solve the partial differential equations $\left(D^3 - 4D^2D^1 + 4DD^{1^2}\right)z = 2\sin(3x+2y)$ (7M)

b) Solve the PDE $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x,0) = 6e^{-3x}$ by the method of variation of parameters. (7M)

Code No: R201206 **R20** SET - 3

I B. Tech II Semester Regular/Supplementary Examinations, August- 2022

MATHEMATICS-III

(Only EEE)

Time: 3 hours Max. Marks: 70

Answer any five Questions one Question from Each Unit
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Unit - I

1 a) In what direction from the point $(1,-2,-1)$ the directional derivative of $\phi = x^2yz + 4xz^2$ is maximum? What is the magnitude of this maximum? (7M)

b) Evaluate $\int_S \bar{F}.\hat{n}\, ds$ where $\bar{F} = z\bar{i} + x\bar{j} - 3y^2z\,\bar{k}$ and 's' is the surface of the cylinder $x^2 + y^2 = 1$ in the first octant between $z = 0$ and $z = 2$. (7M)

Or

2 a) Show that the vector $\left(x^2 - yz\right)\bar{i} + \left(y^2 - zx\right)\bar{j} + \left(z^2 - xy\right)\bar{k}$ is irrotational and find its scalar potential. (7M)

b) If V is the first octant bounded by $y^2 + z^2 = 9$ and the plane $x = 2$ and $\bar{F} = 2x^2\, y\bar{i} - y^2\,\bar{j} + 4xz^2\,\bar{k}$ Then Evaluate $\iint_S \bar{F}.\bar{n}\, ds$ using Gauss-divergence theorem (7M)

Unit - II

3 a) Find the inverse Laplace transform of $\frac{s^3-3s^2+6s-4}{(s^2-2s+2)^2}$ (7M)

b) Evaluate $\int_0^t e^t sin3t\, dt$ using Laplace transform method (7M)

Or

4 a) Find $L\left\{\frac{cos4t\ sin2t}{t}\right\}$ (7M)

b) Find $L^{-1}\left\{\frac{1}{2}\, log\left[\frac{s+1}{s-1}\right]\right\}$ (7M)

Unit - III

5 a) Find the Fourier series of $f(x) = 4 - x^2 \ \ in \ \ -2 < x < 2$ (7M)

b) Using Fourier cosine integral, show that $\frac{\pi}{2}e^{-x} = \int_0^\infty \frac{\cos\lambda x}{\lambda^2+1}d\lambda$ (7M)

Or

6 a) Find the Half range sine of $f(x) = \begin{cases} x & 0 < x < \frac{\pi}{2} \\ (\pi - x) & \frac{\pi}{2} < x < \pi \end{cases}$ (7M)

b) Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2 & if\ |x| < a \\ 0 & if\ |x| > a \end{cases}$ (7M)

Unit - IV

7 a) Solve the PDE $(y^2-z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$ (7M)

b) Find the partial differential equation by eliminating arbitrary function from $f(x^2+y^2,\ x^2-z^2)=0$ (7M)

Or

8 a) Solve the PDE $\frac{p^2}{z^2} = 1 - pq$ (7M)

b) Find the partial differential equation by eliminating arbitrary constants a & b from $z = xy + y\sqrt{x+a} + b$ (7M)

Unit - V

9 a) Solve the partial differential equations $\left(D^2 + DD^1 - 6D^{1^2}\right)z = x + y$ (7M)

b) Solve the PDE $4\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 3z$ and $z(0, y) = e^{-5y}$ by the method of variation of parameters. (7M)

Or

10 a) Solve the wave equation $\frac{\partial^2 y}{\partial x^2} = c^2 \frac{\partial^2 y}{\partial t^2}$ (7M)

Subject to

(i) $y(0,t) = 0$

(ii) $y(\pi,t) = 0$

(iii) $y(x,0) = x, 0 \le x \le \pi$

(iv) $\frac{\partial y}{\partial t}(x,0) = 0, 0 \le x \le \pi$

b) Solve the partial differential equations $\left(D^3 - 3D^2D^1 + 4D^{1^3}\right)z = e^{x+2y}$ (7M)

Code No: R201206 **R20** SET - 4

I B. Tech II Semester Regular/Supplementary Examinations, August - 2022

MATHEMATICS-III

(Only EEE)

Time: 3 hours Max. Marks: 70

Answer any five Questions one Question from Each Unit
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Unit - I

1 a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ in the directional of $2\bar{i} - \bar{j} - 2\bar{k}$ at $(1,-2,-1)$ (7M)

b) Evaluate $\int_V \bar{f}\, dv = xz\bar{i} + 2x\bar{j} - y^2\bar{k}$ over the volume bounded by $x = 0$, $y = 0$, $z = x^2$; $z = 2$, $x = 2$. (7M)

Or

2 a) Find $curl\,\bar{f}$ for $\bar{f} = grad\left(x^3 + y^3 + z^3 - 3xyz\right)$ (7M)

b) Verify the Divergence theorem for $\bar{F} = 4xy\,\bar{i} - y^2\,\bar{j} + xz\,\bar{k}$ over the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$. (7M)

Unit - II

3 a) Find $L\left\{\frac{e^{at} - cos5t}{t}\right\}$ (7M)

b) Find the inverse Laplace transform of $\frac{1+2s}{(s+2)^2(s-1)^2}$ (7M)

Or

4 a) Evaluate $\int_0^t \int_0^t \int_0^t \frac{1}{2} e^{3t} t^2\, dt\, dt\, dt$ using Laplace transform method (7M)

b) Using Laplace transform method, solve the following differential equation $(D^4 - k^4)y = 0$ if $y(0) = 1, y'(0) = 0, y''(0) = 0, y'''(0) = 0$ (7M)

Unit - III

5 a) Find the Half range cosine series of $f(x) = \frac{\pi x}{8}(\pi - x)$ in $[0,\pi]$ (7M)

b) Express the $f(x) = \begin{cases} 1 & if\ 0 < x < \pi \\ 0 & if\ x > \pi \end{cases}$ as a Fourier sine integral (7M)

Or

6 a) Find the Fourier series of $f(x) = \begin{cases} \pi x & 0 < x < 1 \\ 0 & 1 < x < 2 \end{cases}$ (7M)

b) Find the Fourier Cosine transform of the function $f(x) = \begin{cases} \sin ax & if\ x < a \\ 0 & if\ x > a \end{cases}$ (7M)

Unit - IV

7 a) Find the partial differential equation by eliminating arbitrary function from $f(x^2+y^2+z^2,\ z^2-2xy)=0$ (7M)

b) Solve the PDE $x(y-z)p+y(z-x)q=z(x-y)$ (7M)

Or

8 a) Find the partial differential equation by eliminating arbitrary constants a, b & c from $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ (7M)

b) Solve the PDE $p^2+pq=z^2$ (7M)

Unit - V

9 a) Solve the following boundary valve problem $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$ with (7M)

(i) $\frac{\partial u}{\partial x}(0,t)=0$

(ii) $\frac{\partial u}{\partial x}(l,t)=0$

(ii) $u(x,0)=x \quad 0\le x\le l$

b) Solve the partial differential equations $\left(D^2-7DD^1+12D^{1^2}\right)z=e^{x-y}$ (7M)

Or

10 a) Solve the partial differential equations $\left(D^2-DD^1\right)z=\sin x\cos 2y$ (7M)

b) Solve the PDE $\frac{\partial u}{\partial x}+4\frac{\partial u}{\partial y}=0$ and $u(0,y)=8e^{-3y}$ by the method of variation of parameters. (7M)