

**I B. Tech II Semester Regular/Supplementary Examinations, August-2022**  
**MATHEMATICS-III**  
 (Only EEE)

Time: 3 hours

Max. Marks: 70

**Answer any five Questions one Question from Each Unit**  
**All Questions Carry Equal Marks**

**Unit - I**

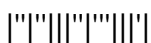
1. a) Prove that  $\nabla\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}$  (7M)
- b) Using Stoke's theorem to evaluate  $\oint_c \vec{A} \cdot d\vec{r}$  where  $\vec{A} = 2y^2\vec{i} + 3x^2\vec{j} - (2x+z)\vec{k}$  and  $c$  is the boundary of the triangle whose vertices are (0, 0, 0), (2, 0, 0), (2, 2, 0). (7M)
- Or
2. a) Prove that  $\text{div}(\vec{A} \times \vec{B}) = \vec{B} \cdot \text{curl} \vec{A} - \vec{A} \cdot \text{curl} \vec{B}$  (7M)
- b) If  $\vec{F} = (x-y)\vec{i} + (x+y)\vec{j}$  Evaluate  $\oint_c \vec{F} \cdot d\vec{r}$  around the curve  $y = x^2$  and  $y^2 = x$  (7M)

**Unit - II**

3. a) Use theorem on Laplace transform of derivatives to evaluate  $L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\}$  (7M)
- b) Find inverse Laplace transform of  $\left\{\frac{s}{(s^2+a^2)^2}\right\}$  using convolution theorem. (7M)
- Or
4. a) Find  $L\{f(t)\}$  where  $f(t)$  is a periodic function of period  $2\pi$  and is given by (7M)
- $$f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & \pi \leq t < 2\pi \end{cases}$$
- b) Find  $L^{-1}\left\{\frac{s^2+2}{s(s^2+4)}\right\}$  (7M)

**Unit - III**

5. a) Find the Fourier series of  $f(x) = \begin{cases} 1 + \frac{2x}{\pi} & \text{if } -\pi \leq x < 0 \\ 1 - \frac{2x}{\pi} & \text{if } 0 \leq x < \pi \end{cases}$  (7M)
- Hence deduce that  $1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$
- b) Find the Fourier transform of  $f(x)$  defined by  $f(x) = \begin{cases} x & \text{if } |x| < a \\ 0 & \text{if } |x| > a \end{cases}$  (7M)
- Or
6. a) Find the Half range cosine series of  $f(x) = e^x$  in  $[0, 1]$  (7M)
- b) Find inverse Fourier sine transform of  $\frac{1}{p}e^{-ap}$  (7M)



## Unit - IV

7. a) Solve the PDE  $p^{1/3} - q^{1/3} = 3x - 2y$  (7M)  
 b) Find the partial differential equation by eliminating arbitrary function from  $ax + by + cz = \phi(x^2 + y^2 + z^2)$  (7M)

Or

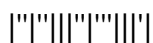
8. a) From the partial differential equation of all planes which are at a constant distance 'a' from the origin. (7M)  
 b) Solve the PDE  $(mz - ny)p + (nx - lz)q = ly - mx$  (7M)

## Unit - V

9. a) Solve the partial differential equation  $(D^2 - 2DD^1 + D^{1^2})z = \sin(2x + 3y)$  (7M)  
 b) Solve the PDE  $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$  and  $u(x, y) = 4e^{-x}$  by the method of variation of parameters. (7M)

Or

10. a) Solve the partial differential equation  $(D^2 - D^{1^2})z = x^2 + y^2$  (7M)  
 b) A rectangular plate with insulated surface is 8 cm. wide. If the temperature along one short edge  $y = 8$  cm. is given by  $100 \sin \frac{\pi x}{8}$ ,  $0 < x < 8$  while the two long edges  $x = 0$  and  $x = 8$  and other edge are kept  $0^\circ\text{C}$ . Find the steady state temperature at any point on the plane. (7M)



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## Unit - I

- 1 a) Show that  $\vec{F} = (e^x z - 2xy) \vec{i} - (x^2 - 1) \vec{j} + (e^x + z) \vec{k}$  is conservative field, Hence Evaluate  $\oint_c \vec{F} \cdot d\vec{r}$  where  $c$  is the end point of the curve  $(0, 1, -1)$  and  $(2, 3, 0)$ . (7M)

- b) Prove that  $\nabla^2 \left( \frac{1}{r} \right) = 0$  (7M)

Or

- 2 a) Prove that  $\nabla \left( e^{r^2} \right) = 2e^{r^2} \vec{r}$  (7M)

- b) Verify Green's theorem for  $\int_c (3x^2 - 8y^2) dx + (4y - 6xy) dy$  where  $c$  is the boundary of the region enclosed by the lines.  $x = 0$   $y = 0$   $x + y = 1$ . (7M)

## Unit - II

- 3 a) Find  $L \left\{ \frac{\sin^2 t}{t^2} \right\}$  (7M)

- b) Using Laplace transform method, solve the following differential equation  $(D^2 + D)x = 2$  if  $x(0) = 3$ ,  $x'(0) = 1$ ,  $x''(0) = -2$  (7M)

Or

- 4 a) Find Laplace transform of unit impulse function or Dirac delta function (7M)

- b) Find inverse Laplace transform of  $\left\{ \frac{1}{(s-2)(s+2)^2} \right\}$  using convolution theorem. (7M)

## Unit - III

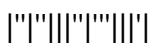
- 5 a) Find the Fourier series of  $f(x) = x + x^2$  in  $(-\pi, \pi)$  (7M)

- b) If  $F(p)$  is the complex Fourier transform of  $f(x)$ , then prove that the complex Fourier transform of  $f(x) \cos ax$  is  $\frac{1}{2} [F(p+a) + F(p-a)]$  (7M)

Or

- 6 a) Find the Half range sine series of  $f(x) = \begin{cases} \frac{1}{4} - x & 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \frac{1}{2} < x < 1 \end{cases}$  (7M)

- b) Find the Fourier sine transform of the function  $f(x) = x$  (7M)



## Unit - IV

- 7 a) Find the partial differential equation by eliminating arbitrary constants a & b from (7M)  

$$z = a \log \left[ \frac{b(y-1)}{1-x} \right]$$
- b) Solve the PDE  $q = px + p^2$  (7M)

Or

- 8 a) Find the partial differential equation by eliminating arbitrary functions from (7M)  

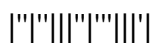
$$z = f(2x+3y) + yg(2x+3y)$$
- b) Solve the PDE  $(x + 2z)p + (4z - y)q = 2x + y$  (7M)

## Unit - V

- 9 a) Solve the partial differential equations  $(D^3 - D^1)z = x^3y^3$  (7M)
- b) Find the temperature  $u(x, t)$  in a homogenous bar of heat conducting method of length 'l' whose ends are kept at  $0^\circ\text{C}$  and whose initial temperature is  $\frac{ax}{l^2}(l-x)$  (7M)

Or

- 10 a) Solve the partial differential equations  $(D^3 - 4D^2D^1 + 4DD^1^2)z = 2 \sin(3x + 2y)$  (7M)
- b) Solve the PDE  $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$ , where  $u(x, 0) = 6e^{-3x}$  by the method of variation of parameters. (7M)



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**Unit - I**

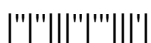
- 1 a) In what direction from the point  $(1, -2, -1)$  the directional derivative of  $\phi = x^2yz + 4xz^2$  is maximum? What is the magnitude of this maximum? (7M)
- b) Evaluate  $\int_s \bar{F} \cdot \hat{n} ds$  where  $\bar{F} = z\bar{i} + x\bar{j} - 3y^2z\bar{k}$  and 's' is the surface of the cylinder  $x^2 + y^2 = 1$  in the first octant between  $z = 0$  and  $z = 2$ . (7M)
- Or
- 2 a) Show that the vector  $(x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$  is irrotational and find its scalar potential. (7M)
- b) If V is the first octant bounded by  $y^2 + z^2 = 9$  and the plane  $x = 2$  and  $\bar{F} = 2x^2 y\bar{i} - y^2 \bar{j} + 4xz^2 \bar{k}$  Then Evaluate  $\iiint_s \bar{F} \cdot \bar{n} ds$  using Gauss-divergence theorem (7M)

**Unit - II**

- 3 a) Find the inverse Laplace transform of  $\frac{s^3 - 3s^2 + 6s - 4}{(s^2 - 2s + 2)^2}$  (7M)
- b) Evaluate  $\int_0^t e^t \sin 3t dt$  using Laplace transform method (7M)
- Or
- 4 a) Find  $L\left\{\frac{\cos 4t \sin 2t}{t}\right\}$  (7M)
- b) Find  $L^{-1}\left\{\frac{1}{2} \log\left[\frac{s+1}{s-1}\right]\right\}$  (7M)

**Unit - III**

- 5 a) Find the Fourier series of  $f(x) = 4 - x^2$  in  $-2 < x < 2$  (7M)
- b) Using Fourier cosine integral, show that  $\frac{\pi}{2} e^{-x} = \int_0^{\infty} \frac{\cos \lambda x}{\lambda^2 + 1} d\lambda$  (7M)
- Or
- 6 a) Find the Half range sine of  $f(x) = \begin{cases} x & 0 < x < \frac{\pi}{2} \\ (\pi - x) & \frac{\pi}{2} < x < \pi \end{cases}$  (7M)
- b) Find the Fourier transform of  $f(x) = \begin{cases} a^2 - x^2 & \text{if } |x| < a \\ 0 & \text{if } |x| > a \end{cases}$  (7M)



**Unit - IV**

7 a) Solve the PDE  $(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$  (7M)

b) Find the partial differential equation by eliminating arbitrary function from  $f(x^2 + y^2, x^2 - z^2) = 0$  (7M)

Or

8 a) Solve the PDE  $\frac{p^2}{z^2} = 1 - pq$  (7M)

b) Find the partial differential equation by eliminating arbitrary constants a & b from  $z = xy + y\sqrt{x+a+b}$  (7M)

**Unit - V**

9 a) Solve the partial differential equations  $(D^2 + DD^1 - 6D^1^2)z = x + y$  (7M)

b) Solve the PDE  $4\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 3z$  and  $z(0, y) = e^{-5y}$  by the method of variation of parameters. (7M)

Or

10 a) Solve the wave equation  $\frac{\partial^2 y}{\partial x^2} = c^2 \frac{\partial^2 y}{\partial t^2}$  (7M)

Subject to

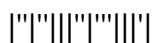
(i)  $y(0, t) = 0$

(ii)  $y(\pi, t) = 0$

(iii)  $y(x, 0) = x, 0 \leq x \leq \pi$

(iv)  $\frac{\partial y}{\partial t}(x, 0) = 0, 0 \leq x \leq \pi$

b) Solve the partial differential equations  $(D^3 - 3D^2D^1 + 4D^1^3)z = e^{x+2y}$  (7M)



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**Unit - I**

- 1 a) Find the directional derivative of  $\phi = x^2yz + 4xz^2$  in the directional of  $2\bar{i} - \bar{j} - 2\bar{k}$  at  $(1, -2, -1)$  (7M)
- b) Evaluate  $\int_V \bar{f} \, dv = xz\bar{i} + 2x\bar{j} - y^2\bar{k}$  over the volume bounded by  $x = 0, y = 0, z = x^2; z = 2, x = 2.$  (7M)

Or

- 2 a) Find  $\text{curl } \bar{f}$  for  $\bar{f} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$  (7M)
- b) Verify the Divergence theorem for  $\bar{F} = 4xy\bar{i} - y^2\bar{j} + xz\bar{k}$  over the cube bounded by  $x = 0, x = 1, y = 0, y = 1, z = 0$  and  $z = 1.$  (7M)

**Unit - II**

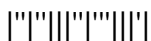
- 3 a) Find  $L\left\{\frac{e^{at} - \cos 5t}{t}\right\}$  (7M)
- b) Find the inverse Laplace transform of  $\frac{1+2s}{(s+2)^2(s-1)^2}$  (7M)
- Or
- 4 a) Evaluate  $\int_0^t \int_0^t \int_0^{\frac{1}{2}} e^{3t} t^2 \, dt \, dt \, dt$  using Laplace transform method (7M)
- b) Using Laplace transform method, solve the following differential equation  $(D^4 - k^4)y = 0$  if  $y(0) = 1, y'(0) = 0, y''(0) = 0, y'''(0) = 0$  (7M)

**Unit - III**

- 5 a) Find the Half range cosine series of  $f(x) = \frac{\pi x}{8}(\pi - x)$  in  $[0, \pi]$  (7M)
- b) Express the  $f(x) = \begin{cases} 1 & \text{if } 0 < x < \pi \\ 0 & \text{if } x > \pi \end{cases}$  as a Fourier sine integral (7M)

Or

- 6 a) Find the Fourier series of  $f(x) = \begin{cases} \pi x & 0 < x < 1 \\ 0 & 1 < x < 2 \end{cases}$  (7M)
- b) Find the Fourier Cosine transform of the function  $f(x) = \begin{cases} \sin ax & \text{if } x < a \\ 0 & \text{if } x > a \end{cases}$  (7M)



## Unit - IV

7 a) Find the partial differential equation by eliminating arbitrary function from  $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$  (7M)

b) Solve the PDE  $x(y - z)p + y(z - x)q = z(x - y)$  (7M)

Or

8 a) Find the partial differential equation by eliminating arbitrary constants a, b & c from  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  (7M)

b) Solve the PDE  $p^2 + pq = z^2$  (7M)

## Unit - V

9 a) Solve the following boundary value problem  $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$  with (7M)

(i)  $\frac{\partial u}{\partial x}(0, t) = 0$

(ii)  $\frac{\partial u}{\partial x}(l, t) = 0$

(ii)  $u(x, 0) = x \quad 0 \leq x \leq l$

b) Solve the partial differential equations  $(D^2 - 7DD^1 + 12D^{1^2})z = e^{x-y}$  (7M)

Or

10 a) Solve the partial differential equations  $(D^2 - DD^1)z = \sin x \cos 2y$  (7M)

b) Solve the PDE  $\frac{\partial u}{\partial x} + 4\frac{\partial u}{\partial y} = 0$  and  $u(0, y) = 8e^{-3y}$  by the method of variation of parameters. (7M)

