I B. Tech I Semester Supplementary Examinations, July/August - 2021 MATHEMATICS-I

(Com. to CE, EEE, ME, ECE, CSE, Chem E, EIE, IT, Auto E, Min E, Pet E, Agri E)
Time: 3 hours

Max. Marks: 75

Answer any five Questions one Question from Each Unit All Questions Carry Equal Marks

- 1. a) Test the convergence of the series $\sum \frac{3^n n!}{n^n}$ (8M)
 - b) Show that for any x > 0, $1 + x < e^x < 1 + xe^x$ (7M)

Or

- 2. a) Test the convergence of the series $\sum \frac{(-1)^n}{n} x^n$ (8M)
 - b) Obtain Taylor's series expansion of sinx in powers of $(x-\pi/2)$ (7M)
- 3. a) Solve the ODE $y(2x^2y + e^x)dx = (e^x + y^3)dy$ (8M)
 - b) The initial value problem governing the current i flowing in series R, L circuit (7M) when voltage v (t) = t is applied is given by $Ri + L\frac{di}{dt} = t$, $t \ge 0$. Find the current i(t) at time t.

Or

- 4. a) Solve the ODE $x^3 \sec^2 y \frac{dy}{dx} + 3x^2 \tan y = \cos x$ (8M)
 - b) Show that the family of curves $\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1$ is self-orthogonal, where λ (7M) is a parameter.
- 5. a) Solve the DE $(D^2 + 2D + 1)y = x \cdot \cos x$ (8M)
 - b) The differential equation for a circuit in which self inductance and capacitance neutralize is $L \frac{d^2q}{dt^2} + \frac{q}{c} = 0$. Find the charge and current, given that I is maximum and charge q is zero at t = 0.

Or

- 6. a) Solve the DE $\frac{d^2y}{dx^2} 4\frac{dy}{dx} + y = \cos x \cos 2x$ (8M)
 - b) Solve the DE $(D^2 + 1)y = \sec^2 x$ by the method of variation of parameters. (7M)

7. a) find the maximum values of the following function using Lagrange's multiplier (8M) method if $u = x^2y^3z^4$ subject to $\phi(x, y, z) = 2x + 3y + 4z - a = 0$

b) Find
$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$$
 using Euler's theorem for $u = \frac{x^2 + y^2}{\sqrt{x} + \sqrt{y}}$ (7M)

Or

8. a) Find
$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$
 using Euler's theorem for $u = \tan^{-1} \left(\frac{y^2}{x} \right)$ (8M)

b) Find the extreme values of $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ (7M)

9. a) Evaluate
$$\int_{0}^{\pi/2} \int_{0}^{a \sin \theta} \int_{0}^{(a^{2}-r^{2})/a} r \, dr \, d\theta \, dz$$
 (8M)

b) Evaluate
$$\int_{0}^{2} \int_{0}^{\sqrt{2x-x^2}} (x^2 + y^2) dx dy$$
 by changing in to polar co-ordinates. (7M)

Or

- 10. a) Evaluate by change of order of integration $\int_{0}^{a} \int_{x}^{a} (x^{2} + y^{2}) dx dy$ (8M)
 - b) Evaluate $\iiint_V xyz \, dx \, dy \, dz$ where v is bounded by the co-ordinate planes and the plane (7M) x + y + z = 1.