

I B. Tech I Semester Regular Examinations, April- 2022
MATHEMATICS-I
 (Com. to All Branches)

Time: 3 hours

Max. Marks: 70

Answer any five Questions one Question from Each Unit
All Questions Carry Equal Marks

Unit - I

1. a) Examine the convergence of $\sum \left[\frac{1.4.7\dots(3n-2)}{3.6.9\dots 3n} \right]^2$. (7M)
- b) If $f(x) = \log x$ and $g(x) = x^2$ in $[a, b]$ with $b > a > 1$, using Cauchy's theorem prove (7M)
 that $\frac{\log b - \log a}{b - a} = \frac{a + b}{2c^2}$.

OR

2. a) Examine the convergence of $\frac{3}{5} - \frac{5}{7} + \frac{7}{10} - \frac{9}{13} + \dots$ (6M)
- b) Find Maclaurin's series expansion of the $f(x) = \sin^2 x$ about $x=1$. (8M)

Unit - II

3. a) Solve $\frac{dy}{dx} - 2\frac{y}{x} - \frac{5x^2}{(x+2)(3-2x)} = 0$. (7M)
- b) Suppose that an object is heated to 300°F and allowed to cool in a room whose air temperature is 80°F , if after 10 minutes the temperature of the object is 250°F , what will be its temperature after 20 minutes. (7M)

OR

4. a) Find the orthogonal trajectories of $r^2 = a \sin 2\theta$. (7M)
- b) Solve $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$. (7M)

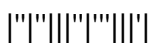
Unit - III

5. a) Solve $(D^2 + 3D + 2)y = e^{-x} + \cos x$. (7M)
- b) Solve $x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} + y = \frac{1}{(1-x)^2}$. (7M)

OR

6. a) In an L-C-R circuit, the charge q on a plate of a condenser is given by (7M)

$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = E \sin pt.$$
 The circuit is tuned to resonance so that $q^2 = 1/LC$. If initially the current I and the charge q be zero, find the current in the circuit.
- b) Solve $(D^2 + 4^2)y = \tan 2x$, by the method of Variation of parameters. (7M)



Unit - IV

7. a) Determine whether the functions $U = \frac{x}{y-z}$, $V = \frac{y}{z-x}$, $W = \frac{z}{x-y}$ are dependent. (7M)
If dependent find the relationship between them.
- b) Expand the function $f(x, y) = xy^2 + \cos(xy)$ in powers of $(x-1)$ and $\left(y - \frac{\pi}{2}\right)$. (7M)

OR

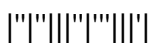
8. a) Find the extreme values of the function $f(x, y) = xy(a-x-y)$. (8M)
- b) Find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ using Euler's theorem for the function $u = \log\left(\frac{x^2 + y^2}{xy}\right)$. (6M)

Unit - V

9. a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} x^2 y^2 (x+y) dy dx$. (7M)
- b) Find the area bounded by the curve $x^2 = 2$, $4y = x^2$ and $y = 4$. (7M)

OR

- 10 a) By transforming into polar coordinates, evaluate $\iint \frac{x^2 y^2}{x^2 + y^2} dx dy$ over the annular region between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$, with $b > a$. (7M)
- b) By changing the order of integration, evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2 - x^2 - y^2} dy dx$. (7M)



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**Unit - I**

1. a) Discuss the convergence of  $\frac{x}{1.3} + \frac{x^2}{3.5} + \frac{x^3}{7.9} + \dots (x > 0)$ . (7M)
- b) Find the region in which  $f(x) = 1 - 4x - x^2$  is increasing and the region in which it is decreasing using Mean Value Theorem. (7M)

OR

2. a) Examine the convergence of  $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$  (7M)
- b) Expand  $\tan^{-1} x$  in powers of  $(x - 1)$  up to fourth degree term. (7M)

**Unit - II**

3. a) Solve  $\cosh x \frac{dy}{dx} + y \sinh x = 2 \cosh^2 x \sinh x$ . (7M)
- b) A metal ball is heated to a temperature of  $100^\circ\text{C}$  and at time  $t = 0$  it is placed in water which is maintained at  $40^\circ\text{C}$ . If the temperature of the ball reduces to  $60^\circ\text{C}$  in 4 minutes, find the time at which the temperature of the ball is  $50^\circ\text{C}$ . (7M)

OR

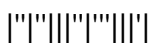
4. a) Find the orthogonal trajectories of the family of curves  $x^{2/3} + y^{2/3} = a^{2/3}$  where 'a' is the parameter. (7M)
- b) Solve  $(xy \sin xy + \cos xy) y dx + (xy \sin xy - \cos xy) x dy = 0$ . (7M)

**Unit - III**

5. a) Solve  $(D^3 - 3D^2 + 4)y = e^{2x} + 6 + 80 \cos 2x$ . (7M)
- b) Solve  $x^2 \frac{d^2 y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$  (7M)

OR

6. a) The charge  $q(t)$  on the capacitor is given by the differential equation (7M)  
 $10 \frac{d^2 q}{dt^2} + 120 \frac{dq}{dt} + 1000q = 17 \sin(2t)$ . At initial time the current is zero and the charge on the capacitor is 0.0005 coulomb. Find the charge on the capacitor for  $t > 0$ .
- b) Solve  $(D^2 + 4)y = \sec 2x$ , by the method of Variation of parameters. (7M)



## Unit - IV

7. a) Determine whether the functions  $U = \frac{x}{y-z}$ ,  $V = \frac{y}{z-x}$ ,  $W = \frac{z}{x-y}$  are dependent. (7M)  
If dependent find the relationship between them.
- b) Expand  $f(x, y) = e^{x+y}$  in the neighborhood of (1, 1). (7M)
- OR
8. a) Find the extreme values of the function  $f(x, y) = x^2y + y^2 + x^4$ . (7M)
- b) Find  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  using Euler's theorem for the function  $u = \log\left(\frac{x^2 + y^2}{x + y}\right)$ . (7M)

## Unit - V

9. a) Evaluate  $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2 - x^2 - y^2} dy dx$ . (7M)
- b) Find the area bounded by the parabola  $y^2 = 4ax$  and its latus rectum. (7M)
- OR
- 10 a) By changing the order of integration, evaluate  $\int_0^3 \int_1^{\sqrt{4-y}} (x+y) dx dy$ . (7M)
- b) Evaluate  $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2 + y^2} dy dx$  by changing into polar coordinates. (7M)



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## MATHEMATICS-I

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**Answer any five Questions one Question from Each Unit**  
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## Unit - I

1. a) Examine the convergence  $\sum \frac{1}{(n^{3/2} + n + 1)}$ . (7M)

b) Verify Lagrange's mean value theorem for  $f(x) = x^3 - x^2 - 5x + 3$  in  $[0,4]$ . (7M)

OR

2. a) Test for convergence of  $1 - \frac{x^2}{2!} + \frac{x^4}{4} - \frac{x^6}{6!} + \dots (0 < x < 1)$ . (7M)

b) Find Taylor's series expansion of the  $f(x) = \cos x$  about  $x = \frac{\pi}{3}$ . (7M)

## Unit - II

3. a) Solve  $(1-x^2)\frac{dy}{dx} + xy = y^3 \sin^{-1} x$ . (7M)

b) If the air is maintained at  $30^\circ\text{C}$  and the temperature of the body cools from  $80^\circ\text{C}$  to  $60^\circ\text{C}$  in 12 minutes, find the temperature of the body after 24 minutes. (7M)

OR

4. a) Find the orthogonal trajectories of  $r = a(1 - \cos \theta)$ . (7M)

b) Solve  $(y \cos x + \sin y + y)dx + (\sin x + x \cos y + x)dy = 0$ . (7M)

## Unit - III

5. a) Solve  $(D^2 - 3D + 2)y = 2x^2$ . (7M)

b) Solve  $(2x+3)^2 \frac{d^2 y}{dx^2} - (2x+3) \frac{dy}{dx} - 12y = 6x$ . (7M)

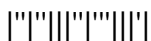
OR

6. a) Solve  $(D^2 - 3D + 2)y = 2x^2$ . (7M)

b) Solve  $(D^2 + 1)y = \operatorname{cosec} x$  by the method of Variation of parameters. (7M)

## Unit - IV

7. a) Check whether  $u = \frac{x^2 - y^2}{x^2 + y^2}$ ,  $v = \frac{2xy}{x^2 + y^2}$  are functionally dependent and find the relation between them. (7M)



- b) Expand  $e^x \cos y$  by Taylor's theorem about the point  $\left(1, \frac{\pi}{4}\right)$  up to the second degree terms. (7M)

OR

8. a) Find the extreme values of the function  $f(x, y) = x^3 + 3xy^2 - 15x^2 + 72x - 15y^2$ . (7M)
- b) Find  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  using Euler's theorem for the function  $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$ . (7M)

**Unit - V**

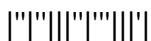
9. a) Evaluate  $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{1+x^2+y^2}$ . (7M)

- b) Find the area bounded by pair of curve  $y = 2 - x$  and  $y^2 = 2(2 - x)$ . (7M)

OR

- 10 a) By changing the order of integration, evaluate  $\int_0^1 \int_1^{2-x} xydx dy$ . (7M)

- b) Using spherical polar coordinates, evaluate  $\iiint xyz dx dy dz$  taken over the volume bounded by the sphere  $x^2 + y^2 + z^2 = a^2$  in the first octant. (7M)



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Unit - I

1. a) Test for convergence of $1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \dots$ (7M)

b) Prove using Mean Value Theorem $|\sin u - \sin v| \leq |u - v|$. (7M)

OR

2. a) Examine the convergence of $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ ($x > 0$) (7M)

b) Find the MacLaurin's expansion of $\log(1 + e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + \dots$ (7M)

Unit - II

3. a) Solve $\frac{dy}{dx} + \frac{y}{x \log x} = \frac{\sin 2x}{\log x}$. (7M)

b) In 20 minutes, a body changes its cools from 80°C to 60°C , and the temperature of air being 40°C . What will be the temperature of the body after 40 minutes from the original? (7M)

OR

4. a) Find the orthogonal trajectories of the family of curves: $r^n = a^n \sin n\theta$. (7M)

b) Solve $(2xy + y - \tan y)dx + (x^2 - x \tan^2 y + \sec^2 y)dy = 0$. (7M)

Unit - III

5. a) Solve $(D^3 - 4D^2 - D + 4)y = e^{3x} \cos 2x$. (7M)

b) Solve $(x + 1)^2 \frac{d^2y}{dx^2} + (x + 1) \frac{dy}{dx} + y = \sin(2 \log(1 + x))$ (7M)

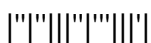
OR

6. a) Solve $(D^2 + D)y = x^2 + 2x + 4$. (7M)

b) Solve $(D^2 - 2D + 1)y = e^x \log x$, by method of variation of parameters. (7M)

Unit - IV

7. a) Check whether $u = x^2 e^{-y} \cosh z$, $v = x^2 e^{-y} \sinh z$, $w = x^2 + y^2 + z^2 - xy - yz - zx$ are functionally dependent. If dependent find the relationship between them. (7M)



b) Expand the function $f(x, y) = \tan^{-1}(xy)$ in powers of $(x - 1)$ and $(y + 1)$. (7M)

OR

8. a) Find the minimum distance from the point $(1, 2, 0)$ on to the cone $z^2 = x^2 + y^2$. (7M)

b) Find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ using Euler's theorem for the function $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$. (7M)

Unit - V

9. a) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dx dy dz$. (7M)

b) Find the area lying between the circle $x^2 + y^2 = a^2$ and the plane $x + y = a$ in the first quadrant. (7M)

OR

10 a) By changing the order of integration, evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x+y) dx dy$. (7M)

b) Using spherical polar coordinates, evaluate $\iiint \frac{xyz dx dy dz}{\sqrt{x^2+y^2+z^2}}$ taken over the volume (7M)

Bounded by the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant.

